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BOOK REVIEW

The Mathematical Theory of Permanent Progressive Water-Waves. By H. Okamoto & M. Shoji. World Scientific, 2001. 228 pp. ISBN 9810244495. £29.

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The theory of water waves is an important topic in fluid mechanics. Many results have been obtained over the last 200 years. These include linear solutions, the pioneering nonlinear works of Stokes, Rayleigh and Korteweg and de Vries, weakly nonlinear theories and rigourous analytical results. Since 1970, a large number of accurate nonlinear computations have been performed. The numerical results have uncovered new branches of solutions and prompted further asymptotic and analytical investigations. Well-known textbooks on water waves are *Linear and Nonlinear Waves* by G. B. Whitham (Wiley 1974) and *Waves in Fluids* by M. J. Lighthill (Cambridge University Press, 1978). The recent book by J. Billingham & A. C. King (*Wave Motion*, Cambridge University Press, 2001) covers many interesting aspects of wave propagation and in particular water waves.

The present book is concerned with the mathematical theory of permanent progressive water waves. The word 'mathematical' indicates that little attention is devoted to the physical motivation behind the various approximations used in the modelling and to the question of whether or not the waves described can be observed in nature or in experiments. Similarly almost no stability analysis of the solutions is presented. One reason is that a complete overview of the theory of water waves cannot be achieved in 215 pages. This problem of space is corrected by an extensive bibliography at the end of the book.

Chapter 1 introduces the problem of two-dimensional periodic gravity capillary waves propagating at a constant velocity at the surface of a fluid bounded below by a horizontal flat bottom. The kinematic and dynamic boundary conditions are derived. It is shown how the problem can be simplified by using the potential function and the streamfunction as independent variables. Boundary integral equation formulations using Hilbert transforms are derived. Waves of small amplitude are studied as a bifurcation from the trivial solution with a flat free surface.

In Chapter 2, the effect of gravity is neglected and pure capillary waves are considered. The exact solutions of Crapper and Kinnnersley are reviewed and interesting questions about uniqueness are addressed.

Chapter 3 contains a nice presentation of the theory of pure gravity waves (i.e. waves in the absence of surface tension). The Stokes' expansion is reviewed. The existence theorems of Krasovskii, Keady & Norbury and Amick, Fraenkel & Toland are among the theoretical results presented. Garabedian's theory on symmetry is also covered. Some of the proofs are presented. Special attention is devoted to the highest gravity wave. Over the years many highly accurate numerical methods have been developed to calculate such waves. Only one numerical method is presented in the book, namely that of H. Yamada. This is a surprising choice since there are easier and more efficient methods (most of them are covered in the bibliography).

Chapters 4 and 5 deal with gravity capillary waves. This problem is more complicated than the previous ones because there is an infinite number of families of solutions. This non-uniqueness was first discovered by Wilton at the beginning of the 20th century. In the last 25 years, various families of solutions have been explored systematically by several investigators. The authors present numerical results obtained by using a spectral-collocation method (the details of the numerical procedure are presented at the end of chapter 4). The various branches are calculated by using path-continuation. The results are displayed in bifurcation diagrams.

Chapter 6 is concerned with waves with negative surface tension! As the authors recognize themselves, it is difficult to find a physical application. Even if we accept the problem as an interesting mathematical exercise, the numerical results presented are not very convincing. There are spurious oscillations on the profile of figure 6.3. It is surprising that the authors did not improve these numerical results by using a different numerical method (for example a boundary integral equation method).

In Chapter 7, the theory is extended by relaxing the condition of the irrotationality of the flow. General equations are derived using a pseudo potential function. Numerical results are presented in the particular case of constant vorticity. The numerical results are interesting but cannot cover in 8 pages all the various types of nonlinear waves which have been computed in recent years.

In Chapter 8 another extension is considered by taking into account the motion in the upper fluid. The book concludes with a very short introduction to solitary waves.

In conclusion, the book covers interesting topics which complement those covered in standard texts. At the end of each chapter there are a few exercises to put into practice the ideas presented. The authors are careful about their numerical experiments and present clearly the limitations of their approaches and the remaining open questions. The book should be of interest to scientists interested in the mathematical theory of water waves. In addition it is likely to stimulate further work on the subject.

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